11.39. **Model:** Use the relationship between a conservative force and potential energy.
Visualize: Please refer to Figure P11.39. We will obtain U as a function of x and F_x as a function of x by using the calculus techniques of integration and differentiation.
Solve: **(a)** For the interval $0 \text{ m} < x < 0.5 \text{ m}$, $F_x = (4x) \text{ N}$, where x is in meters. This means

$$\frac{dU}{dx} = -F_x = -4x \Rightarrow U = -2x^2 + C_1 = -2x^2$$

where we have used $U = 0 \text{ J}$ at $x = 0 \text{ m}$ to obtain $C_1 = 0$. For the interval $0.5 \text{ m} < x < 1 \text{ m}$, $F_x = (-4x + 4) \text{ N}$. Likewise,

$$\frac{dU}{dx} = 4x - 4 \Rightarrow U = 2x^2 - 4x + C_2$$

Since U should be continuous at the junction, we have the continuity condition

$$(-2x^2)_{x=0.5 \text{ m}} = (2x^2 - 4x + C_2)_{x=0.5 \text{ m}} \Rightarrow -0.5 = 0.5 - 2 + C_2 \Rightarrow C_2 = 1$$

U remains constant for $x \geq 1$ m.
(b) For the interval $0 \text{ m} < x < 0.5 \text{ m}$, $U = +4x$, and for the interval $0.5 \text{ m} < x < 1.0 \text{ m}$, $U = -4x + 4$, where x is in meters. The derivatives give $F_x = -4 \text{ N}$ and $F_x = +4 \text{ N}$, respectively. The slope is zero for $x \geq 1$ m.